

Lattice QCD study for relation between confinement and chiral symmetry breaking on temporally odd-number lattice

Takahiro M. Doi^{*}, Hideo Suganuma

*Department of Physics & Division of Physics and Astronomy, Graduate School of Science,
Kyoto University, Kitashirakawaoiwake, Sakyo, Kyoto 606-8502, Japan
E-mail: doi@ruby.scphys.kyoto-u.ac.jp*

Takumi Iritani

High Energy Accelerator Research Organization (KEK), Tsukuba, Ibaraki 305-0801, Japan

We investigate the contribution from each Dirac modes to the Polyakov loop based on a gauge-invariant analytical relation connecting the Polyakov loop and the Dirac modes on a temporally odd-number lattice, where the temporal lattice size is odd, with the normal (nontwisted) periodic boundary condition. The dumping factor in the relation plays crucial role for the negligible contribution of low-lying Dirac modes to the Polyakov loop. The zero-value of the Polyakov loop in the confinement phase is due to the “positive/negative symmetry” of the Dirac-mode contribution to the Polyakov loop. In the deconfinement phase, there is no such symmetry.

*XV International Conference on Hadron Spectroscopy-Hadron 2013
4-8 November 2013
Nara, Japan*

^{*}Speaker.

1. Introduction

Color confinement and chiral symmetry breaking have been investigated as interesting non-perturbative phenomena in low-energy QCD in many analytical and numerical studies. However, their properties are not sufficiently understood directly from QCD. The Polyakov loop is an order parameter for quark confinement [1]. At the quenched level, the Polyakov loop is the exact order parameter for quark confinement, and its expectation value is zero in confinement phase and is nonzero in deconfinement phase. Also, its fluctuation is recently found to be important in the QCD phase transition [2]. As for the chiral symmetry, low-lying Dirac modes are essential for chiral symmetry breaking in QCD, according to the Banks-Casher relation [3].

Not only the properties of confinement and chiral symmetry breaking in QCD but also their relation is an interesting challenging subject [4, 5]. From many analytical and numerical studies, it is suggested that confinement and chiral symmetry breaking are strongly correlated [6, 7]. However, we showed analytically and numerically that low-lying Dirac modes have little contribution to the Polyakov loop and that there is no one-to-one correspondence between confinement and chiral symmetry breaking in QCD [8, 9, 10].

In this study, we discuss the relation between confinement and chiral symmetry breaking based on an analytical relation between the Polyakov loop and Dirac modes on temporally odd-number lattice, with the normal (nontwisted) periodic boundary condition [9, 10]. We investigate each Dirac-mode contribution to the Polyakov loop in both confinement and deconfinement phases.

2. Dirac modes in lattice QCD

In this section, we review the Dirac operator, its eigenvalues and its eigenmodes (Dirac modes) in $SU(N_c)$ lattice QCD [8]. We use a standard square lattice with spacing a , and the notation of sites $s = (s_1, s_2, s_3, s_4)$ ($s_\mu = 1, 2, \dots, N_\mu$), and link-variables $U_\mu(s) = e^{iagA_\mu(s)}$ with gauge fields $A_\mu(s) \in su(N_c)$ and gauge coupling g . In lattice QCD, the Dirac operator $\mathcal{D} = \gamma_\mu D_\mu$ is given by

$$\mathcal{D}_{s,s'} = \frac{1}{2a} \sum_{\mu=1}^4 \gamma_\mu [U_\mu(s) \delta_{s+\hat{\mu},s'} - U_{-\mu}(s) \delta_{s-\hat{\mu},s'}], \quad (2.1)$$

with $U_{-\mu}(s) \equiv U_\mu^\dagger(s - \hat{\mu})$. Here, $\hat{\mu}$ is the unit vector in direction μ in the lattice unit. In this paper, we define all the γ -matrices to be hermite as $\gamma_\mu^\dagger = \gamma_\mu$. Since the Dirac operator is anti-hermite in this definition of γ_μ , the Dirac eigenvalue equation is expressed as

$$\mathcal{D}|n\rangle = i\lambda_n|n\rangle \quad (2.2)$$

with the Dirac eigenvalue $i\lambda_n$ ($\lambda_n \in \mathbf{R}$) and the Dirac eigenstate $|n\rangle$.

3. An analytical relation between the Polyakov loop and Dirac modes on temporally odd-number lattice

We consider a temporally odd-number lattice, where the temporal lattice size N_4 is odd, with the normal (nontwisted) periodic boundary condition in both temporal and spatial directions. The

spatial lattice size $N_{1\sim 3}(> N_4)$ is taken to be even. Using Elitzur's theorem, we derive a relation connecting the Polyakov loop and the Dirac modes [9, 10],

$$\langle L_P \rangle = \frac{(2ai)^{N_4-1}}{12V} \sum_n \lambda_n^{N_4-1} \langle n | \hat{U}_4 | n \rangle, \quad (3.1)$$

where the link-variable operator $\hat{U}_{\pm\mu}$ is defined by the matrix element

$$\langle s | \hat{U}_{\pm\mu} | s' \rangle = U_{\pm\mu}(s) \delta_{s\pm\hat{\mu}, s'}. \quad (3.2)$$

In this derivation, we use the fact that any gauge-invariant quantity cannot be composed by the product of odd-number N_4 link-variables [9, 10], except for the Polyakov loop. This is a Dirac spectral representation of the Polyakov loop, and is valid on the temporally odd-number lattice. Using this relation (3.1), we can investigate each Dirac-mode contribution to the Polyakov loop individually and discuss the relation between confinement and chiral symmetry breaking in QCD.

4. Modified KS formalism for temporally odd-number lattice

The Dirac operator \not{D} has a large dimension of $(4 \times N_c \times V)^2$, so that the numerical cost for solving the Dirac eigenvalue equation is quite huge. This numerical cost can be partially reduced using the Kogut-Susskind (KS) formalism [1, 8, 11]. However, the original KS formalism can be applied only to the “even lattice” where all the lattice sizes N_μ are even number. In this section, we show brief introduction of the modified KS formalism [10] applicable to the odd-number lattice.

We consider the temporally odd-number lattice, and introduce a matrix

$$M(s) \equiv \gamma_1^{s_1} \gamma_2^{s_2} \gamma_3^{s_3} \gamma_4^{s_1+s_2+s_3}. \quad (4.1)$$

Using the matrix $M(s)$ and taking the Dirac representation, we can spin-diagonalize the Dirac operator \not{D} in the case of the temporally odd-number lattice,

$$\sum_\mu M^\dagger(s) \gamma_\mu D_\mu M(s + \hat{\mu}) = \text{diag}(\eta_\mu D_\mu, \eta_\mu D_\mu, -\eta_\mu D_\mu, -\eta_\mu D_\mu), \quad (4.2)$$

where $\eta_\mu D_\mu$ is the KS Dirac operator given by

$$(\eta_\mu D_\mu)_{ss'} = \frac{1}{2a} \sum_{\mu=1}^4 \eta_\mu(s) [U_\mu(s) \delta_{s+\hat{\mu}, s'} - U_{-\mu}(s) \delta_{s-\hat{\mu}, s'}]. \quad (4.3)$$

Here, $\eta_\mu(s)$ is the staggered phase: $\eta_1(s) \equiv 1$, $\eta_\mu(s) \equiv (-1)^{s_1+\dots+s_{\mu-1}}$ ($\mu \geq 2$). Thus, all the eigenvalues $i\lambda_n$ can be obtained by solving the reduced Dirac eigenvalue equation,

$$\eta_\mu D_\mu |n\rangle = i\lambda_n |n\rangle. \quad (4.4)$$

5. Numerical analysis for each Dirac-mode contribution to the Polyakov loop

Using the modified KS formalism, Eq.(3.1) is rewritten as

$$\langle L_P \rangle = \frac{(2ai)^{N_4-1}}{3V} \sum_n \lambda_n^{N_4-1} \langle n | \hat{U}_4 | n \rangle. \quad (5.1)$$

Note that the (modified) KS formalism is an exact method for diagonalizing the Dirac operator and is not an approximation, so that Eqs.(3.1) and (5.1) are completely equivalent.

We numerically calculate each Dirac-mode contribution to the Polyakov loop, i.e., the matrix elements $\langle n|\hat{U}_4|n\rangle$ and $\lambda_n^{N_4-1}\langle n|\hat{U}_4|n\rangle$ in the sum of the relation (5.1). We perform SU(3) lattice QCD Monte Carlo simulations with the standard plaquette action at the quenched level in both cases of confinement and deconfinement phases. For the confinement phase, we use $10^3 \times 5$ lattice with $\beta \equiv 2N_c/g^2 = 5.6$ (i.e., $a \simeq 0.25$ fm), corresponding to $T \equiv 1/(N_4 a) \simeq 160$ MeV. For the deconfinement phase, we use $10^3 \times 3$ lattice with $\beta = 5.7$ (i.e., $a \simeq 0.20$ fm), corresponding to $T \equiv 1/(N_4 a) \simeq 330$ MeV. For each phase, we use 20 gauge configurations, which are taken every 500 sweeps after the thermalization of 5,000 sweeps.

As the numerical result, we find that the relation (5.1) is almost exact and low-lying Dirac modes have little contribution to the Polyakov loop for each gauge configuration in both confinement and deconfinement phases [10]. Thus, we can discuss each Dirac-mode contribution to the Polyakov loop even for one gauge configuration.

In the confinement phase, we show in Fig.1 each Dirac-mode contribution to the Polyakov loop $\lambda_n^{N_4-1}\langle n|\hat{U}_4|n\rangle$ as the function of Dirac eigenvalue λ_n . From Fig. 1, we can confirm that low-lying Dirac modes have little contribution to the Polyakov loop. All the sum of these quantities $\lambda_n^{N_4-1}\langle n|\hat{U}_4|n\rangle$ is zero, which leads to the vanishing Polyakov loop in the confinement phase. As a remarkable fact, the zero value of the Polyakov loop is due to the “positive/negative symmetry” of real and imaginary parts of $\lambda_n^{N_4-1}\langle n|\hat{U}_4|n\rangle$, as shown in Fig.1.

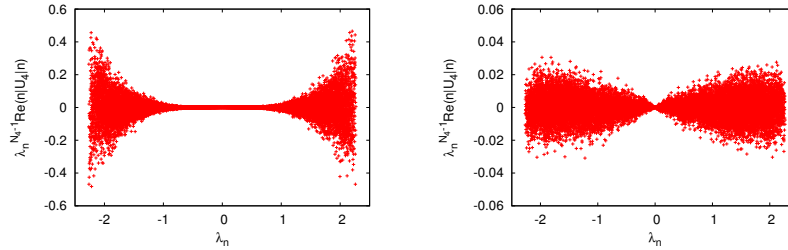


Figure 1: Lattice QCD results for each Dirac-mode contribution to the Polyakov loop, $\lambda_n^{N_4-1}\langle n|\hat{U}_4|n\rangle$, as the function of the Dirac eigenvalue λ_n in the lattice unit, on $10^3 \times 5$ with $\beta = 5.6$, for one gauge configuration in the confinement phase. The left figure shows the real part and the right figure the imaginary part.

In the deconfinement phase, we show in Fig.2 the real part of the matrix elements $\langle n|\hat{U}_4|n\rangle$ and each Dirac-mode contribution to the Polyakov loop, $\lambda_n^{N_4-1}\langle n|\hat{U}_4|n\rangle$, as the function of Dirac eigenvalue λ_n . In this configuration, the expectation value of the Polyakov loop is real, and the behavior of the imaginary part of these quantities is similar to that in the confinement phase. There is no “positive/negative symmetry” in the Dirac-mode distribution of $\text{Re}(n|\hat{U}_4|n)$, and all the sum of these quantities $\lambda_n^{N_4-1}\text{Re}(n|\hat{U}_4|n)$ is nonzero, which gives a nonzero value of the Polyakov loop in the deconfinement phase. The signs of the contribution from infrared Dirac modes and ultraviolet Dirac modes are different. Although the matrix elements $\text{Re}(n|\hat{U}_4|n)$ have a peak in the small Dirac-mode region, the contribution from this region to the Polyakov loop is very small, because of the dumping factor $\lambda_n^{N_4-1}$. Thus, the factor $\lambda_n^{N_4-1}$ plays a crucial role in RHS of Eq.(5.1).

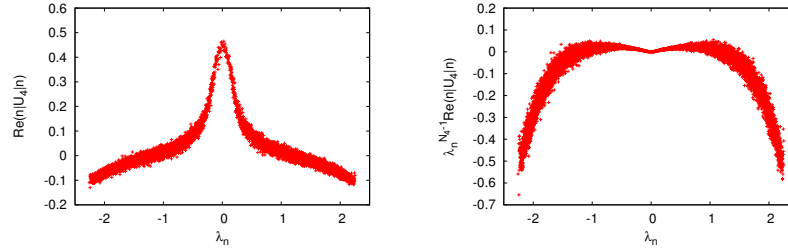


Figure 2: Lattice QCD results for the real part of the matrix element $\lambda_n^{N_4-1} \text{Re}(n|\hat{U}_4|n)$ and each Dirac-mode contribution to the Polyakov loop, $\lambda_n^{N_4-1} \text{Re}(n|\hat{U}_4|n)$, as the function of the Dirac eigenvalue λ_n in the lattice unit, on $10^3 \times 3$ with $\beta = 5.7$, for one gauge configuration in the deconfinement phase.

6. Summary and concluding remarks

In this study we have investigated each Dirac-mode contribution to the Polyakov loop based on the relation connecting the Polyakov loop and the Dirac modes on temporally odd-number lattice, with the normal (nontwisted) periodic boundary condition. In both confinement and deconfinement phases, low-lying Dirac modes have little contribution to the Polyakov loop because of the dumping factor $\lambda_n^{N_4-1}$ in RHS of Eq.(5.1). Also, the zero-value of the Polyakov loop in confinement phase is due to the “positive/negative symmetry” of the Dirac-mode matrix elements $(n|\hat{U}_4|n)$. In the deconfinement phase, there is no such symmetry.

Acknowledgements

H.S. and T.I. are supported in part by the Grant for Scientific Research [(C) No.23540306, E01:21105006, No.21674002] from the Ministry of Education, Science and Technology of Japan. The lattice QCD calculation has been done on NEC-SX8R at Osaka University.

References

- [1] H.J. Rothe, *Lattice Gauge Theories*, (World Scientific, 2012), and its references.
- [2] P.M. Lo, B. Friman, O. Kaczmarek, K. Redlich and C. Sasaki, *Phys. Rev. D* **88**, (2013) 014506; *Phys. Rev. D* **88**, (2013) 074502.
- [3] T. Banks and A. Casher, *Nucl. Phys.* **B169** (1980) 103.
- [4] H. Suganuma, S. Sasaki and H. Toki, *Nucl. Phys.* **B435** (1995) 207.
- [5] C. Gattringer, *Phys. Rev. Lett.* **97** (2006) 032003.
F. Bruckmann, C. Gattringer and C. Hagen, *Phys. Lett.* **B647** (2007) 56.
- [6] O. Miyamura, *Phys. Lett.* **B353** (1995) 91; R.M. Woloshyn, *Phys. Rev. D* **51** (1995) 6411.
- [7] F. Karsch, *Lect. Notes Phys.* **583** (2002) 209, and its references.
- [8] S. Gongyo, T. Iritani and H. Suganuma, *Phys. Rev. D* **86** (2012) 03451; T. Iritani and H. Suganuma, [arXiv:1305.4049 [hep-lat]]; T. Iritani et al., *PoS (Confinement X)* (2013) 053.
- [9] H. Suganuma, T.M. Doi and T. Iritani, *PoS (Lattice 2013)* (2013) 374;
Eur. Phys. J. Web of Conferences (ICNFP2013) (2014); *PoS (QCD-TNT-III)* (2014) 042.
- [10] T.M. Doi, H. Suganuma and T. Iritani, *PoS (Lattice 2013)* (2013) 375.
- [11] J.B. Kogut and L. Susskind, *Phys. Rev. D* **11** (1975) 395.